Math 10B with Professor Stankova
Quiz 13; Tuesday, 4/24/2018
Section #203; Time: 9:30 AM
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Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** If $\det(A) = 0$, we need to use Gaussian elimination to determine if $A\vec{v} = \vec{0}$ has 0 or ∞ solutions.

> **Solution:** The system of equations $A\vec{v} = \vec{0}$ always has the trivial solution as a solution which means that it has at least one solution. That means that if $\det(A) = 0$, it has infinitely many solutions.

2. **TRUE** False If $\det(A) = 0$, then 0 is an eigenvalue for A.

> **Solution:** $\lambda = 0$ solves $\det(A - 0I) = \det(A - 0) = \det(A) = 0$ so $\lambda = 0$ is an eigenvalue.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (6 points) Let $A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. Calculate A^{-1} using Gaussian elimination.

> **Solution:** Using Gaussian elimination
>
> $$\left(\begin{array}{ccc|ccc} 0 & -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{I \leftrightarrow II} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{III-I} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$
>
> $$\xrightarrow{I+II} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & -1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right) \xrightarrow{I-2III, II-2III} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -2 \\ 0 & -1 & 0 & 1 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$
>
> $$\xrightarrow{II\cdot(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -2 \\ 0 & 1 & 0 & -1 & -2 & 2 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$
>
> So the inverse is $\begin{pmatrix} 1 & 3 & -2 \\ -1 & -2 & 2 \\ 0 & -1 & 1 \end{pmatrix}$.

(b) (1 point) Let $\vec{y} = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$. Find the matrix B that such that $\vec{y}' = B\vec{y}$ given

$$\begin{cases} y_1'(t) = y_1(t) + 2y_2(t) \\ y_2'(t) = y_1(t) \end{cases}$$

Solution: $B = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$

(c) (3 points) Find the eigenvalues and eigenvectors of the matrix B found above.

Solution: We have to look at $\det(B - \lambda I) = (1-\lambda)(-\lambda) - 2 = \lambda^2 - \lambda - 2 = (\lambda - 2)(\lambda + 1)$. Thus the eigenvalues are $\lambda = -1, 2$. For $\lambda = -1$, we have $B - \lambda I = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ and so an eigenvector is $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$. For $\lambda = 2$, we have $B - \lambda I = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}$ so an eigenvector is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.